Comment on "Noncommutativity as a Possible Origin of the Ultrahigh-energy Cosmic Ray and the TeV Photon Paradoxes"

Cosmas Zachos

High Energy Physics Division, Argonne National Laboratory, Argonne, IL 60439-4815, USA
zachos@anl.gov

Abstract

A Lorentz-noninvariant modification of the kinematic dispersion law was proposed in Ref [1], claimed to be derivable from from q-deformed noncommutative theory, and argued to evade ultrahigh energy threshold anomalies (trans-GKZ-cutoff cosmic rays and TeV-photons) by raising the respective thresholds. It is pointed out that such dispersion laws do not follow from deformed oscillator systems, and the proposed dispersion law is invalidated by tachyonic propagation, as well as photon instability, in addition to the process considered.

Reported anomalous cosmic threshold events serve to constrain hypothesized violations of Lorentz invariance [2, 3, 4, 5]. E.g., observations of roughly isotropic cosmic rays with $p \geq 4 \times 10^{19}$eV, which are above the Greisen-Zatsepin-Kuz'min (GZK) threshold, or else, detection of up to 20 TeV photons from the active galaxy (blazar) Markarian 501. The GZK threshold for $p + \gamma \rightarrow p + \pi$, ensures absorption, within about 50 Mpc, of cosmic rays such as protons with energies above $p_{GKZ} = ((m_p + m_\pi)^2 - m_p^2)/(4k)$, where k is an active energy of available cosmic microwave background photons, $\sim 10^{-3} - 10^{-4} eV$. Similarly, reported 20 TeV photons from Markarian 501 (at a distance of 157 Mpc, much longer than the mean free path for photons cut off by collisions with infrared photons to pair-produce, $\gamma + \gamma \rightarrow e^+ + e^-$) should have been absorbed: the cutoff can effectively set in at about $p^\gamma = m_e^2/k$, where k is an active energy of universal IR background photons, $\sim 10^{-1} eV$, thought to result out of star formation in the early universe [4]. These thresholds might be evaded by small deformations of Lorentz kinematics, for instance by raising them to higher energies which allow for the reportedly observed events [2, 3, 4, 5].

One such kinematic energy-momentum dispersion law was proposed in ref [1], namely,

$$E = \frac{\omega}{2}\left(\left[\frac{\sqrt{m^2+p^2}}{\omega} - \frac{1}{2}\right]_q + \left[\frac{\sqrt{m^2+p^2}}{\omega} + \frac{1}{2}\right]_q\right) = \frac{\omega}{2}\left[\frac{\sqrt{m^2+p^2}}{\omega/2}\right]_{\sqrt{q}}, \tag{1}$$

where

$$[x]_q \equiv \frac{q^x - q^{-x}}{q - q^{-1}} \quad . \tag{2}$$

The deformation parameter q is taken to be close to unity, so that, for $\kappa \equiv \ln q$, the standard Lorentz kinematic dispersion is modified,

$$E = \frac{\omega}{2}\frac{\sinh(\kappa\sqrt{m^2+p^2}/\omega)}{\sinh(\kappa/2)} = \sqrt{m^2+p^2}\left(1 + \frac{\kappa^2}{24}\left(\frac{4(m^2+p^2)}{\omega^2} - 1\right)\right) + O(\kappa^4). \tag{3}$$

The other parameter, $\omega/2$, is the universal energy scale needed for a nonlinear law, suggested in ref [1] to be a fraction of an MeV. (NB. The actual rest mass of the particle is finitely "renormalized" to $E(p=0)$, i.e., $M = \omega \sinh(\kappa m/\omega)/(2 \sinh(\kappa/2))$.)

This law, for both fermions and bosons, was argued in ref [1] to follow from standard deformed oscillator systems [6]. It is pointed out that this law cannot follow from deformed oscillator systems, and that it does not address the threshold anomaly paradoxes satisfactorily.

The starting point of ref [1] is the standard q-fermion oscillator hamiltonian [7],

$$\hat{H}_q = \frac{\omega}{2}\left([\hat{N}]_q - [\widehat{1-N}]_q\right), \tag{4}$$

where the number operator $\hat{N}$ counts creation and annihilation operators for self-excuding fermions (or deformed fermions, likewise self-excluding [7]), and has eigenvalues 1 or 0 only. It is thus idempotent, $\hat{N}^2 = \hat{N}$, and, as a consequence, $q^{\hat{N}} = 1 + \hat{N}(q-1)$. Consequently, it has been broadly appreciated, for a while [8], that the q-hamiltonian for fermions (4) is *no different* than the free fermion oscillator hamiltonian limit of it ($q = 1$), viz. $\hat{H} = \omega(\hat{N} - 1/2)$, despite appearances to the contrary.

Thus, no nontrivial inference may be drawn from the q-fermion hamiltonian (4), let alone the proposed violation of Lorentz invariance.

Beyond fermions, ref [1] also addresses q-bosons, through the popular Biedenharn-McFarlane hamiltonian, for bosonic q-oscillators commuting with each other, and integer-valued $\hat{N}$,

$$\hat{H}_q = \frac{\omega}{2}\left([\hat{N}]_q + [\hat{N}+1]_q\right) = \frac{\omega}{2}\left(\left[\frac{\hat{h}}{\omega} - \frac{1}{2}\right]_q + \left[\frac{\hat{h}}{\omega} + \frac{1}{2}\right]_q\right), \tag{5}$$

where $\hat{h}$ is the undeformed free hamiltonian. This is interacting [9], not free, as its spectrum is not linearly spaced. Conceivably, it specifies some nonlinear medium of sorts, and leads to a modification of the Planck distribution [10] (which has not been fully evaluated analytically; nevertheless, its critical properties are different than those of a boson gas, [11, 12]). Such distributions might allow constraining the parameters κ and ω through comparison with cosmic microwave background spectral data, but would evidently provide poor constraints. (They might also affect the average impacted photon energies assumed in GKZ threshold analyses, but not drastically.)

In second-quantizing noncommutative scalar fields [13], one normally assembles an infinity of oscillators of different momenta p, each of which, in fact, could be related to the above, through suitable deformations [14]. One then works out the dependence of the momentum-dependent coefficient of the plain oscillators in the free hamiltonian to obtain a dispersion law $E(p)$ in such media, and to then investigate its Lorentz-violating implications [13]. By contrast, ref [1] proposes to merely substitute the eigenvalue $E = \sqrt{m^2 + p^2}$ for the free hamiltonian $\hat{h}$ inside the *single-mode* (first quantized) interacting (5) to produce the dispersion law (1), claimed to hold for this medium, i.e., to have the nonlinear interaction provide a deformation of the kinematics for *all* momenta! This is plainly unwarranted.

In conclusion, for both fermions and bosons, deformation hamiltonians do not lead to the modified kinematic dispersion (1,3).

As an aside, rather than stretch for supporting arguments and motivation, one might alternatively consider this dispersion as a mere ad hoc heuristic: i.e., simply postulate the dispersion (1,3), as a modifier of the index of refraction p/E in a hypothetical medium, and ask if it raises the cosmic thresholds to address the more severe multi-TeV-photon potential paradox. It is argued that it does not succeed.

The dispersion (3) is plagued by a tachyonic region, since the group velocity for $m = 0$ is

$$\frac{dE}{dp} = \frac{\kappa \cosh(\kappa p/\omega)}{2 \sinh(\kappa/2)} = 1 + \frac{\kappa^2}{2} \left(\frac{p^2}{\omega^2} - \frac{1}{12} \right) + O(\kappa^4), \tag{6}$$

tachyonic for, e.g., small κ and for sufficiently energetic photons, $p \geq \omega/\sqrt{12}$, well within the parameter ranges utilized in ref [1]. The pitfalls of such superluminal propagation are unforgiving to causality [15].

The index of refraction is

$$\frac{p}{E} = \frac{2p\ \sinh(\kappa/2)}{\omega\ \sinh(\kappa p/\omega)}\ , \tag{7}$$

smaller than the conventional undeformed value ($\kappa = 0$), since sinh (and cosh) are positive definite and convex for positive arguments.

Furthermore, the photon in this picture is severely kinematically unstable to decay, $\gamma \to e^+ + e^-$ [2]. At threshold for decay to an electron-positron pair, the leptons have half the momentum p of the photon, so from energy conservation and (3), photons of momentum p decay if

$$\sinh\left(\frac{p\kappa}{\omega}\right) \geq 2 \sinh\left(\frac{\kappa}{\omega}\sqrt{m_e^2 + \frac{p^2}{4}}\right). \tag{8}$$

The corresponding threshold condition for $\gamma + \gamma \to e^+ + e^-$ contemplated in ref [1], in the frame where the microwave and IR background is essentialy isotropic, should amount to

$$\sinh\left(\frac{p\kappa}{\omega}\right) + \sinh\left(\frac{k\kappa}{\omega}\right) \geq 2 \sinh\left(\frac{\kappa}{\omega}\sqrt{m_e^2 + \frac{(p-k)^2}{4}}\right). \tag{9}$$

The undeformed limit ($\kappa = 0$) threshold for absorption is $p^\gamma = m_e^2/k$, where $k \sim 10^{-1}eV$. For such $p^\gamma \gg m_e \gg k$, both processes above converge to each other and both absorb multi-TeV photons by the convexity of the hyperbolic sine functions for positive arguments, $\sinh(2x) > 2\sinh x$, so such photons do not survive. (Ref [1] employs a κ/ω ranging from $10^{-9}TeV^{-1}$, effectively linearizing the formulas, to large values, for which the leading exponentials in both of the above formulas dominate the hyperbolic functions, and hence photons of momentum larger than $2\frac{\omega}{\kappa}\ \ln 2 \sim 1.39\frac{\omega}{\kappa}$ are absorbed.)

Note Added in Proof: F Stecker kindly points out that the extragalactic 20 TeV photon "anomaly" of ref [4] is itself unwarranted, and may be explained away through proper consideration of intergalactic absorption [16], thereby providing accurate confirmation of Lorentz invariance. The purported observational absence or displacement of the GKZ cutoff has also been seriously controverted [17].

Acknowledgement This work was supported by the US Department of Energy, Division of High Energy Physics, Contract W-31-109-ENG-38.

References

[1] S-X Chen and Z-Y Yang, *Mod Phys Lett* **A18** (2003) 2913

[2] S Coleman and S Glashow, *Phys Lett* **B405** (1997) 249-252; *Phys Rev* **D59** (1999) 116008

[3] R Aloisio, P Blasi, P Ghia, and A Grillo, *Phys Rev* **D62** (2000) 053010;
G Amelino-Camelia and T Piran, *Phys Rev* **D64** (2001) 036005;
T Tamaki, T Harada, U Miyamoto, and T Torii, *Phys Rev* **D65** (2002) 083003

[4] R Protheroe and H Meyer, *Phys Lett* **B493** (2000) 1-6

[5] T Kifune, *Ast J Lett* **518** (1999) L21-L24

[6] G Iwata, *Prog Theo Phys* **VI** (1951) 524-528

[7] Y Ng, *J Phys* **A23** (1990) 1023

[8] S Jing and J Xu, *J Phys* **A24** (1991) L891-L894

[9] S Vokos and C Zachos, *Mod Phys Lett* **A9** (1994) 1-9 [hep-th/9305067]

[10] M. Martín-Delgado, *J Phys* **A24** (1991) L1285-L1291;
P Nešković and B Urošević, *Int J Mod Phys* **A7** (1992) 3379

[11] M R-Monteiro, I Roditi, and L Rodrigues, *Phys Lett* **A188** (1994) 11-15.

[12] M Salerno, *Phys Rev* **E50** (1994) 4528-4530.

[13] J Carmona, J Cortés, J Gamboa, and F Méndez, *JHEP* **0303** (2003) 058 [hep-th/0301248];
Phys Lett **B565** (2003) 222-228

[14] A Polychronakos, *Mod Phys Lett* **A5** (1990) 23252334;
X-C Song, *J Phys* **A23** (1990) L821

[15] R Lehnert, *Phys Rev* **D68** (2003) 085003;
A Kostelecký and Lehnert, *Phys Rev* **D63** (2001) 065008

[16] F Stecker, *Astropart Phys* **20** (2003) 85;
A Konopelko et al, *Astrophys J* **597** (2003) 851-859;
F Stecker and S Glashow, *Astropart Phys* **16** (2001) 97

[17] J Bahcall and E Waxman, *Phys Lett* **B556** (2003) 1